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# Adaptive size-independent control of UD and BD vehicle convoys with partial measurement based on constant distance plan

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**ABSTRACT**

The adaptive size-independent consensus problem of uni-directional (UD) and bi-directional (BD) decentralized large-scale vehicle convoys with uncertain dynamics has been investigated in this research work. The constant distance plan (CDP) is employed to adjust the distances between successive vehicles. We assume that only relative displacement information between adjacent vehicles is accessible (partial measurement) and other information such as relative velocity and acceleration are not provided. The stability of the convoy can be performed by the analysis of each couple of consecutive vehicles. The main objective is to design an adaptive size-independent control protocol maintaining internal and string stability based on CDP with only partial measurement. Appropriate adaptive rules are derived to estimate the uncertain dynamics by utilizing only relative displacement. It will be proved that the presented adaptive protocol assures both internal stability (asymptotic stability of closed-loop convoy) and string stability (tracking error attenuation) of large-scale decentralized UD and BD convoys under the CDP. Simulations demonstrate the efficiency of the presented control framework.

## 1. Introduction

In recent few decades, the traffic jam problem has led to fuel loss, environmental pollution, and safety problems [1, 2]. By improving the intelligence level of vehicles and ways (roads and highways), we can promote highway capacity and safety. This idea has led to the invention of the concept of automated highway systems (AHSs) [3]. An AHS is believed as an effective direction for the problems arising from traffic congestion [4, 5]. For AHSs, considering longitudinal vehicle control from the perspective of one-dimensional vehicle convoy control is an attractive research topic [6, 7].

In a convoy of connected vehicles, vehicles are in communication with their neighbors which is called vehicle-to-vehicle (V2V) communication. V2V communication may be done through data transmission by wireless systems (absolute position, velocity, and acceleration) or by measuring data by onboard sensors (relative displacement, velocity, and acceleration). If a vehicle employs the onboard sensors to measure relative information toward its front and rear vehicles, the control strategy will be adaptive cruise control (ACC) [8]. If communication is done through wireless systems to obtain information from other vehicles, the control strategy will be cooperative adaptive cruise control (CACC) [9]. By implementing the CACC in a convoy, the road

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capacity will increase compared with the ACC [10]. The most important purpose in controlling a vehicle convoy is organizing the motion of all vehicles in an equal velocity with as small as possible inter-vehicle distance [11].

The driving levels of a fully automated vehicle is divided in two types: 1) high-level control system and 2) low-level control system. The proper values of acceleration and velocity are calculated by high-level control system and required signals for the throttling and braking systems to implement these commands are generated by the low-level control [12, 13]. Every automatic vehicle and its low-level control system have a role of node dynamics for high-level control system [14]. Two main plans are utilized to set the distance between vehicles. The constant distance plan (CDP) means that the inter-vehicle distances are constant without depending on the velocity [15] and the constant headway plan (CHP) means that the distance changes and increases by increasing the velocity and decreases by decreasing the velocity [16].

A vehicle convoy is string stable if any changes in the leader velocity are attenuated by following vehicles downstream the convoy. If the velocity changes of the leader are amplified by downstream vehicles, the convoy is called string unstable [17]. In [18, 19] it is proved that the linear control protocols are not able to achieve the string stability of uni-directional (UD) and bi-directional (BD) decentralized vehicle convoys with CDP.

Concerns about the internal stability (asymptotic stability of the closed-loop convoy) and string stability (tracking error attenuation) of uncertain large-scale vehicle convoys have been an avenue of research in recent years. Few research studies deal with the stability analysis of uncertain vehicle convoys.

A third-order adaptive distributed sliding mode controller (SMC) is introduced in [20] to achieve the internal stability. In this paper, the string stability has not been studied and the control and adaptive rules employ the relative displacement, velocity and acceleration between vehicles. The BD coupled SMC presented in [21] is unable to achieve the string stability in transient motion before reaching the sliding surface. The adaptive controls proposed in [22] guarantee the string stability if the persistently exciting condition be satisfied. A BD Second-order SMC is presented in [23, 24] to assure the internal and string stability. A third-order adaptive backstepping consensus is presented in [25] to assure the internal stability despite of the existence of unknown dynamics. In [26], an adaptive consensus is devised for a non-identical uncertain BD vehicle convoy by considering the engine saturation to maintain the cohesiveness. A finite-time SMC theory, which can robustly assure zero tracking error and string stability is devised by considering communication interruptions by [27].

We cannot apply the most of previous approaches to large-scale convoys. The adaptive centralized method proposed in [28] can be applied only on small size convoys. The linear BD control method presented in [29] is not scalable. Meaning that by increasing the number of following vehicles, the stability margin of the closed-loop system tends to zero causing the instability problems. The linear control approaches cannot be applied to the large-scale decentralized BD convoys under CDP. Since the control parameters depend on the minimum eigenvalues of the network matrix [30]. On the other hand, any control method whose gains depend on the network matrix (or the number of following vehicles) is not practical on size-changing vehicle convoys [14, 22, 31-33]. In some common maneuvers (merge, split, add and leave), the number of followers will change. Consequently, the dimension of the network matrix varies and consequently, the control gains should be changed.

Motivated by the previous researches, we will introduce a novel robust adaptive consensus method for decentralized uncertain large-scale UD and BD vehicle convoys with CDP to achieve the following objectives. 1) Internal stability: we will prove that under the proposed adaptive robust controller, both UD and BD convoys are internal stable. 2) Scalability: it will be shown that control parameters are not dependent on the convoy size and the minimum eigenvalues of the matrix network. So that, the controller is scalable and robust against the size-varying of the convoy. 3) String stability: linear decentralized controllers cannot achieve the string stability of convoys with CDP. We will prove that the presented adaptive robust controller assures the string stability and 4) using minimum accessible information: each vehicle has access only to relative displacement with respect to the front and rear vehicles (BD topology) and front vehicle (UD topology).

An adaptive control that only uses the relative displacement information is designed. Since the relative velocity is not available, it is replaced with the output of a first-order filter. Appropriate adaptive rules are defined to estimate the uncertain dynamics. For both UD and BD convoys, the transfer function between the tracking errors of consecutive vehicles is derived, and by carrying out the stability analyses in the Laplace domain, the





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required constraints on control gains guaranteeing the internal stability are calculated. In both approaches, it will be shown that the control and adaptation gains are not dependent on the communication topology and the number of the following vehicles. After that, by doing the error propagation analyses in the frequency domain, appropriate constraints on control gains guaranteeing string stability will be derived.

The most important novelties of this research work are: 1) internal stability despite of existence of unknown dynamics by utilizing only relative displacement information, 2) string stability of decentralized UD and BD convoys with constant distance plan, and 3) robustness against the size-varying of the convoy.

This paper is structured as follows. Mathematical preliminaries are discussed in section 2. Internal stability analysis for UD and BD topologies is performed in section 3. Section 4 studies the string stability. Simulation studies are provided in section 5. Finally, this paper is concluded by section 6.

## 2. Preliminaries

The longitudinal uni-directional (UD) and bi-directional (BD) vehicle convoys are depicted in Fig. 1.

The one-dimensional (1-D) dynamical model of the leader is described through the following second-order equation.

$$\ddot{p}_0 = \sigma_0 \tag{1}$$

where $p_0$ denotes the position and $\sigma_0 = a_0(t)$ is the leader unknown acceleration. For *i*-th vehicle, the 1-D dynamics is represented by the following dynamics.

$$\ddot{p}_i = u_i + \sigma_i, \qquad (i = 1,2,\ldots,N) \tag{2}$$

Where $p_i$ is position, $u_i = \bar{u}_i/m_i$ is control input and $m_i$ is the mass of *i-th* vehicle. $\sigma_i = -\left(R_{r,i} + R_{a,i}\right)/m_i$, where $R_{r,i}$ is the rolling resistance and $R_{a,i}$ is the air drag force of *i*-th vehicle. $R_{a,i} = \rho A_{c,i} c_D v_i^2/2$ where $\rho$ is air density, $A_{c,i}$ is cross-sectional area and $c_D$ is the drag coefficient. Moreover, $R_{r,i} = k_{0,i} + k_{1,i} q_i$ where $k_{0,i}$ and $k_{1,i}$ represent the coefficients of rolling resistance [13] and $q_i$ is the *i*-th vehicle velocity.

**Definition 1**. The displacement tracking error of *i*-th vehicle is defined as:

$$e_i = p_i - p_0 + \sum_{j=0}^{i-1}\left(l_j + R_{\min}\right) \tag{3}$$

where $l_j$ denotes the length of *j-th* vehicle and $R_{min}$ is the minimum safe inter-vehicle distance.

**Definition 2**. The relative displacement and velocity errors are defined as:

UD topology: (4)

$$\begin{cases} e_{p,i} = p_i - p_{i-1} + r_{i,i-1} \\ e_{q,i} = q_i - q_{i-1} \end{cases}$$

BD topology: (5)

$$\begin{cases} e_{p,i} = \left(p_i - p_{i-1} + r_{i,i-1}\right) + \left(p_i - p_{i+1} - r_{i,i+1}\right) = \\ 2p_i - p_{i-1} - p_{i+1} + r_{i,i-1} - r_{i,i+1} \\ e_{q,i} = \left(q_i - q_{i-1}\right) + \left(q_i - q_{i+1}\right) = \\ 2q_i - q_{i-1} - q_{i+1} \end{cases}$$

where $r_{i,i-1} = l_{i-1} + R_{min}$ and $r_{i,i+1} = l_{i+1} + R_{min}$.

**Definition 3** [13]. A vehicle convoy is string stable if any changes in the leader velocity are attenuated by following vehicles downstream the convoy.

**Lemma 1**. The following equation describes the relation between the displacement tracking error and the relative displacement error:

UD topology: (6)

$$e_{p,i} = e_i - e_{i-1} \Rightarrow e_{q,i} = \dot{e}_i - \dot{e}_{i-1}$$

BD topology: (7)

$$e_{p,i} = 2e_i - e_{i-1} - e_{i+1} \Rightarrow e_{q,i} = 2\dot{e}_i - \dot{e}_{i-1} - \dot{e}_{i+1}$$

**Proof**. For BD topology, by replacing the following equations in (5),

$$\begin{aligned} p_i &= e_i + p_0 - \sum_{j=0}^{i-1}\left(l_j + R_{\min}\right), \\ p_{i-1} &= e_{i-1} + p_0 - \sum_{j=0}^{i-2}\left(l_j + R_{\min}\right), \\ p_{i+1} &= e_{i+1} + p_0 - \sum_{j=0}^{i}\left(l_j + R_{\min}\right) \end{aligned} \tag{8}$$

and knowing that $r_{i,i-1} = l_{i-1} + R_{\min}$, $r_{i,i+1} = l_i + R_{\min}$, we will have:





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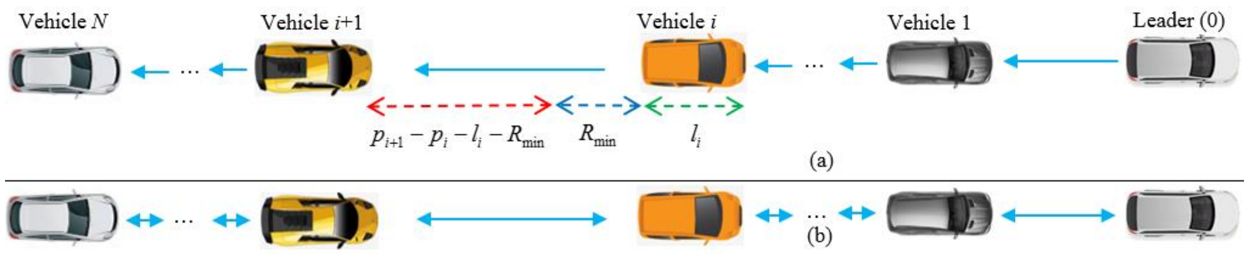


**Fig. 1.** Two longitudinal vehicle convoys, (a): uni-directional and (b): bi-directional.

$$e_{p,i}=\begin{pmatrix} e_i+p_0-\sum_{j=0}^{i-1}\left(l_j+R_{\min}\right)-e_{i-1}-p_0+ \\ \sum_{j=0}^{i-2}\left(l_j+R_{\min}\right)+l_{i-1}+R_{\min} \end{pmatrix}+ \begin{pmatrix} e_i+p_0-\sum_{j=0}^{i-1}\left(l_j+R_{\min}\right)-e_{i+1}-p_0+ \\ \sum_{j=0}^{i}\left(l_j+R_{\min}\right)-l_i-R_{\min} \end{pmatrix}= 2e_i-e_{i-1}-e_{i+1} \tag{9}$$

By doing a similar procedure for UD topology, (6) will be obtained.

## 3. Robust adaptive size-independent consensus problem

This section studies the internal stability of decentralized large-scale uncertain UD and BD vehicle convoys based on CDP. Only relative displacement measurement between adjacent vehicles is available and none of the vehicles have access to relative velocity information than their neighbors.

### 3.1. Decentralized large-scale UD vehicle convoys

Based on the CDP by employing only relative displacement measurement between vehicles $i$ and $i$-1, the following control algorithm is defined as the high-level control of the $i$-th vehicle $(i = 1,\ \dots,\ N)$.

$$u_i=-a\left(\varepsilon_i+e_{p,i}\right)+\hat{\sigma}_{0i}-\hat{\sigma}_i \tag{10}$$

where $a$ is a constant positive gain, $\hat{\sigma}_{0i}$ is the estimate of lead vehicle acceleration calculated by the *i-th* vehicle, $\hat{\sigma}_i$ is the estimate of $\sigma_i$ and $\varepsilon_i$ has the following dynamics.

$$\varepsilon_i=\delta_i+\beta_2 e_{p,i},\quad \dot{\delta}_i=-\beta_1\varepsilon_i \tag{11}$$

where $\beta_1$ and $\beta_2$ are positive gains and $\delta_i$ denotes the intermediate dynamics. The relative velocity of successive vehicles is not accessible. So that, we utilize $\varepsilon_i$ instead of $e_{q,i}$ in the control law (10). The uncertain parameters $\sigma_{0i}$ and $\sigma_i$ are estimated by the following adaptation rules.

$$\begin{aligned} \dot{\hat{\sigma}}_{0i}&=\alpha_2\left(\varepsilon_i-e_{p,i}\right)-\alpha_1 e_{q,i}, \\ \dot{\hat{\sigma}}_i&=\alpha_2\left(e_{p,i}-\varepsilon_i\right)+\alpha_1 e_{q,i} \end{aligned} \tag{12}$$

where $\alpha_1$ and $\alpha_2$ are positive values. The relative velocity of adjacent vehicles is not provided and consequently, the adaptive laws (12) cannot be applied. Therefore, we define the following intermediate parameters.

$$\varphi_{0i}=\hat{\sigma}_{0i}+\alpha_1 e_{p,i},\qquad \varphi_i=\hat{\sigma}_i-\alpha_1 e_{p,i} \tag{13}$$

Merging (12) with time derivation of (13) will result to:

$$\dot{\varphi}_{0i}=\alpha_2\left(\varepsilon_i-e_{p,i}\right),\quad \dot{\varphi}_i=\alpha_2\left(e_{p,i}-\varepsilon_i\right) \tag{14}$$

By knowing the initial values $\varphi_{0i}(0)$ and $\varphi_i(0)$, their instant values are obtained by (14). Afterwards, $\hat{\sigma}_{0i}$ and $\hat{\sigma}_i$ will be determined by (13). In other words, $\hat{\sigma}_{0i}$ and $\hat{\sigma}_i$ are determined by (13) and (14) instead of (12). From (3), we have:

$$\dot{p}_i=\dot{e}_i+q_0\Rightarrow\ddot{p}_i=\ddot{e}_i \tag{15}$$

By combining (2), (6), (10) and (15), the error dynamics of each vehicle will be obtained as:

$$\ddot{e}_i=-a\left(\varepsilon_i+e_i-e_{i-1}\right)+\tilde{\sigma}_{0i}-\tilde{\sigma}_i \tag{16}$$

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Taking Laplace transform of (11), (12) and (16) yields:

$$\varepsilon_i(s)=\frac{s\beta_2}{s+\beta_1}\left(E_i(s)-E_{i-1}(s)\right) \tag{17}$$

$$\tilde{\sigma}_{0,i}(s)=-\frac{\alpha_1 s^2+\left(\alpha_2+\alpha_1\beta_1-\alpha_2\beta_2\right)s+\beta_1\alpha_2}{s\left(s+\beta_1\right)}\times\left(E_i(s)-E_{i-1}(s)\right) \tag{18}$$

$$\tilde{\sigma}_i(s)=\frac{\alpha_1 s^2+\left(\alpha_2+\alpha_1\beta_1-\alpha_2\beta_2\right)s+\beta_1\alpha_2}{s\left(s+\beta_1\right)}\times\left(E_i(s)-E_{i-1}(s)\right) \tag{19}$$

$$s^2E_i(s)=-a\left(\varepsilon_i(s)+E_i(s)-E_{i-1}(s)\right)+\tilde{\sigma}_{0i}(s)-\tilde{\sigma}_i(s) \tag{20}$$

Replacing (17)-(19) in (20) and simplifying the resultant equation will yield:

$$s^3\left(s+\beta_1\right)E_i(s)=-\begin{pmatrix}\left(a\beta_2+a+2\alpha_1\right)s^2+\\ 2\left(a\beta_1+2\alpha_2+2\alpha_1\beta_1-2\alpha_2\beta_2\right)s+\\ 2\beta_1\alpha_2\end{pmatrix}\times\left(E_i(s)-E_{i-1}(s)\right) \tag{21}$$

By simplifying (21), one can write:

$$E_i(s)=\Gamma_i(s)E_{i-1}(s) \tag{22}$$

$$\Gamma_i(s)=\frac{\left(a\beta_2+a+2\alpha_1\right)s^2+2\left(a\beta_1+2\alpha_2+2\alpha_1\beta_1-2\alpha_2\beta_2\right)s+2\beta_1\alpha_2}{s^4+\beta_1 s^3+\left(a\beta_2+a+2\alpha_1\right)s^2+2\left(a\beta_1+2\alpha_2+2\alpha_1\beta_1-2\alpha_2\beta_2\right)s+2\beta_1\alpha_2}$$

**Theorem 1**. Under the control and adaptive rules (10) and (12), the uncertain large-scale UD vehicle convoy described by (1) and (2) is internal stable if the following condition is fulfilled.

$$\max\left\{\frac{\alpha_1\beta_1}{2\alpha_2}+1,\frac{2\beta_1\alpha_2}{a\left(a+2\alpha_1\right)}\right\}<\beta_2<\frac{2\alpha_1\beta_1}{a\beta_1+2\alpha_2}+1 \tag{23}$$

**Proof**. The Routh–Hurwitz stability criterion is employed for the denominator of the transfer function $\Gamma_i(s)$. The internal stability is assured if the sign of the elements in the first column of the Routh array preserves. The Routh array is in the following form.

$$\begin{array}{cccc} s^4 & 1 & a\beta_2+a+2\alpha_1 & 2\beta_1\alpha_2 \\ s^3 & \beta_1 & 2\left(a\beta_1+2\alpha_2+2\alpha_1\beta_1-2\alpha_2\beta_2\right) & 0 \\ s^2 & \bar{A}_1 & 2\beta_1^2\alpha_2 & 0 \\ s^1 & \bar{A}_2 & 0 & 0 \\ s^0 & 2\beta_1^2\alpha_2 & 0 & 0 \end{array} \qquad \begin{array}{l} \bar{A}_1=2\alpha_2\left(\beta_2-1\right)-\alpha_1\beta_1 \\ \bar{A}_2=\left(a\beta_1^2\beta_2\left(a+2\alpha_1\right)-2\beta_1^3\alpha_2\right)+ \\ 2\alpha_2\left(\left(a\beta_1+2\alpha_2\right)\left(1-\beta_2\right)+2\alpha_1\beta_1\right)\left(\beta_2-1\right) \end{array} \tag{24}$$

It is obvious that if $\beta_2>\frac{\alpha_1\beta_1}{2\alpha_2}+1$, $\bar{A}_1>0$. Moreover, the first and second terms of $\bar{A}_2$ are positive under the following conditions.

$$a\beta_1^2\beta_2\left(a+2\alpha_1\right)-2\beta_1^3\alpha_2>0\Rightarrow\beta_2>\frac{2\beta_1\alpha_2}{a\left(a+2\alpha_1\right)}$$
$$\left(a\beta_1+2\alpha_2\right)\left(1-\beta_2\right)+2\alpha_1\beta_1>0\Rightarrow\beta_2<\frac{2\alpha_1\beta_1}{a\beta_1+2\alpha_2}+1 \tag{25}$$

Therefore, under the condition (24), the internal stability is assured. For example, if we choose $\beta_2=1$, (23) will be simplified to $\beta_1<\frac{a(a+2\alpha_1)}{2\alpha_2}$.

### 3.2. Decentralized large-scale BD vehicle convoys

This subsection investigates the internal stability of the uncertain large-scale BD convoys described by (1) and (2). We assume that *i-th* vehicle has access only to the relative displacement than vehicles *i*-1 and *i*+1. To achieve a robust adaptive protocol, the high-level control law (10) is considered for each vehicle. For BD topology, $\varepsilon_i,\hat{\sigma}_{0i}$ and $\hat{\sigma}_i$ are obtained from (11) and (12) by calculating $e_{p,i}$ from (5). It should be noted that due to inaccessibility of $e_{q,i}$, the estimates $\hat{\sigma}_{0i}$ and $\hat{\sigma}_i$ are calculated by (13) in which the intermediate parameters $\varphi_{0i}$ and $\varphi_i$ are obtained by the adaptive rules (14).

In BD vehicular platoons, the closed-loop dynamics of the *i-th* vehicle will be as follows:

$$\ddot{e}_i=-a\left(\varepsilon_i+2e_i-e_{i-1}-e_{i+1}\right)+\tilde{\sigma}_{0i}-\tilde{\sigma}_i \tag{26}$$

For BD platoon one can write:

$$\varepsilon_i(s)=\frac{s\beta_2}{s+\beta_1}\left(2E_i(s)-E_{i-1}(s)-E_{i+1}(s)\right) \tag{27}$$

$$\tilde{\sigma}_{0,i}(s)=-\frac{\alpha_1 s^2+\left(\alpha_2+\alpha_1\beta_1-\alpha_2\beta_2\right)s+\beta_1\alpha_2}{s\left(s+\beta_1\right)}\times\left(2E_i(s)-E_{i-1}(s)-E_{i+1}(s)\right) \tag{28}$$

$$\tilde{\sigma}_i(s)=\frac{\alpha_1 s^2+\left(\alpha_2+\alpha_1\beta_1-\alpha_2\beta_2\right)s+\beta_1\alpha_2}{s\left(s+\beta_1\right)}\times\left(2E_i(s)-E_{i-1}(s)-E_{i+1}(s)\right) \tag{29}$$



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$$s^2 E_i(s) = -a\begin{pmatrix} \varepsilon_i(s) + 2E_i(s) \\ -E_{i-1}(s) - E_{i+1}(s) \end{pmatrix} + \tilde{\sigma}_{0i}(s) - \tilde{\sigma}_i(s) \tag{30}$$

By replacing (27)-(29) in (30) and simplifying the resultant equation, one can write:

$$s^3(s+\beta_1)E_i(s) = -\left(\left(a\beta_2 + a + 2\alpha_1\right)s^2 + 2\begin{pmatrix} a\beta_1 + 2\alpha_2 + \\ 2\alpha_1\beta_1 - 2\alpha_2\beta_2 \end{pmatrix}s + 2\beta_1\alpha_2\right)\left(2E_i(s) - E_{i-1}(s) - E_{i+1}(s)\right) \tag{31}$$

Eq. can be written as follows.

$$E_i(s) = \bar{\Gamma}_i(s)\left(E_{i-1}(s) + E_{i+1}(s)\right) \tag{32}$$

$$\bar{\Gamma}_i(s) = \frac{\left(a\beta_2 + a + 2\alpha_1\right)s^2 + 2\left(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2\right)s + 2\beta_1\alpha_2}{s^4 + \beta_1 s^3 + 2\left(a\beta_2 + a + 2\alpha_1\right)s^2 + 4\left(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2\right)s + 4\beta_1\alpha_2}$$

**Theorem 2**. Under the following conditions, the BD convoy will be internal stable.

$$\beta_2 > \frac{2\beta_1\alpha_2}{a\beta_1 + 4\alpha_2} + 1 \tag{33}$$

$$\frac{4\alpha_2(\beta_2+1)}{3a + 4\alpha_1 + 2\alpha_2} < \beta_1 < \frac{2\alpha_2(\beta_2-1)}{a + 2\alpha_1} \tag{34}$$

**Proof**. From (32), we conclude that the internal stability is achieved if all poles of the transfer function $\bar{\Gamma}_i(s)$ have negative real parts. We employ the Routh-Hurwitz method on the denominator of $\bar{\Gamma}_i(s)$ according to the following table.

(35)

| | | | |
|---|---|---|---|
| $s^4$ | $1$ | $2(a\beta_2 + a + 2\alpha_1)$ | $4\beta_1\alpha_2$ |
| $s^3$ | $\beta_1$ | $4(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2)$ | $0$ |
| $s^2$ | $\bar{A}_1$ | $4\beta_1^2\alpha_2$ | $0$ |
| $s^1$ | $\bar{A}_2$ | $0$ | $0$ |
| $s^0$ | $4\beta_1^2\alpha_2$ | $0$ | $0$ |

$$\bar{A}_1 = (a\beta_1 + 4\alpha_2)(\beta_2 - 1) - 2\beta_1\alpha_2$$

$$\begin{aligned}\bar{A}_2 = {} & a\beta_1^2\beta_2(a+2\alpha_1) + 2a\alpha_2\beta_1\beta_2(\beta_2-1) + \\ & + 4\alpha_2\beta_1(a+2\alpha_1)(\beta_2-1) - 8\alpha_2^2(\beta_2^2-1) \\ & - \beta_1^2(a+2\alpha_1)(a+2\alpha_2) \\ & + 2\alpha_2\beta_1(\beta_2-1)(a+2\alpha_2)\end{aligned}$$

According to the above table, the internal stability is assured if all elements of the first column be positive. By doing mathematical manipulations, we can verify that under the condition (33), $\bar{A}_1$ is positive and under the conditions of (34), $\bar{A}_2 > 0$. Therefore, the BD convoy in the presence of uncertain dynamics will be internal stable and the proof is complete.

**Notes.**

1. According to theorems 1 and 2, for UD and BD topologies, the control and adaptive gains are not dependent on the convoy size. Therefore, the control protocol **Error! Reference source not found.** is size-independent, robust against changing the number of vehicles. In other words, the proposed control methods are easily scalable as they are independent of the convoy size.
2. The adaptive controller (10) only utilize relative displacement information between adjacent vehicles. Therefore, by refraining from using velocity and acceleration sensors, GPS systems, and communication tools, the undesirable effects time delay and data loss will be decreased dramatically.

## 4. String Stability based on constant distance plan

In this section, the string stability of UD and BD convoys based on CDP is studied. A vehicle convoy is string stable if any changes in the leader velocity are attenuated by following vehicles downstream the convoy.

### 4.1. String stability of UD network

A vehicle convoy is string stable if $|E_{i-1}(j\omega)|/|E_i(j\omega)| < 1$. According to (22), the UD vehicle convoy is string stable if $|\Gamma_i(j\omega)| < 1$.

**Theorem 3**. The uncertain large-scale decentralized UD convoy described by (1) and (2) is string stable if we have:

$$\beta_2 > \frac{\beta_1(a + 2\alpha_1)}{2\alpha_2} \tag{36}$$

**Proof**. Let us define that

$$\Gamma_i(s) = \frac{As^2 + Bs + C}{s^4 + \beta_1 s^3 + As^2 + Bs + C} = \frac{N(s)}{D(s)} \tag{37}$$
$$A = a\beta_2 + a + 2\alpha_1, B = 2\left(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2\right), C = 2\beta_1\alpha_2$$

If $|D(j\omega)|^2 - |N(j\omega)|^2 > 0$, then $|\Gamma_i(j\omega)| < 1$ where:





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$$|N(j\omega)|^2 = A^2\omega^4 + \left(B^2 - 2AC\right)\omega^2 + C^2 \tag{38}$$
$$|D(j\omega)|^2 = \omega^8 + \left(\beta_1^2 - 2A\right)\omega^6 + \left(A^2 + 2C - 2B\beta_1\right)\omega^4 + \left(B^2 - 2AC\right)\omega^2 + C^2$$

$$|D(j\omega)|^2 - |N(j\omega)|^2 > 0 \Rightarrow \omega^4 + \left(\beta_1^2 - 2A\right)\omega^2 + 2\left(C - B\beta_1\right) > 0 \tag{39}$$

As proved in [19], the region of low-frequency is the most determinative in verifying the string stability of the convoys. Therefore, the free term $2(C - B\beta_1)$ is dominant compared with other terms. So that, if $C - B\beta_1 > 0$, the string stability is achieved which results to:

$$2\beta_1\alpha_2 - \left(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2\right)\beta_1 > 0$$
$$\Rightarrow \beta_2 > \frac{\beta_1\left(a + 2\alpha_1\right)}{2\alpha_2}$$

and the proof is complete.

### 4.2. String stability of BD network

Similar to previous subsection, the following theorem is presented for BD convoys.

**Theorem 4**. The uncertain large-scale decentralized BD convoy described by (1) and (2) is string stable if we have:

$$\beta_2 > \frac{\beta_1\left(a + 2\alpha_1\right)}{2\alpha_2} \tag{40}$$

**Proof**. Under the condition $\|\bar{\Gamma}_i(j\omega)\| = \|\bar{N}(j\omega)/\bar{D}(j\omega)\| < 0.5$, a BD vehicle network will be string stable [22]. We define that:

$$\bar{\Gamma}_i(s) = \frac{\bar{A}s^2 + \bar{B}s + \bar{C}}{s^4 + \beta_1 s^3 + 2\bar{A}s^2 + 2\bar{B}s + 2\bar{C}} = \frac{\bar{N}(s)}{\bar{D}(s)} \tag{41}$$
$$\bar{A} = a\beta_2 + a + 2\alpha_1, \bar{B} = 2\begin{pmatrix} a\beta_1 + 2\alpha_2 + \\ 2\alpha_1\beta_1 - 2\alpha_2\beta_2 \end{pmatrix}, \bar{C} = 2\beta_1\alpha_2$$

If $|\bar{D}(j\omega)|^2 - 4|\bar{N}(j\omega)|^2 > 0$, then $|\bar{\Gamma}_i(j\omega)| < 0.5$ where:

$$|\bar{N}(j\omega)|^2 = \bar{A}^2\omega^4 + \left(\bar{B}^2 - 2\bar{A}\bar{C}\right)\omega^2 + \bar{C}^2 \tag{42}$$
$$|\bar{D}(j\omega)|^2 = \omega^8 + \left(\beta_1^2 - 4\bar{A}\right)\omega^6 + 4\left(\bar{A}^2 + \bar{C} - \bar{B}\beta_1\right)\omega^4 + 4\left(\bar{B}^2 - 2\bar{A}\bar{C}\right)\omega^2 + 4\bar{C}^2$$

$$|\bar{D}(j\omega)|^2 - 4|\bar{N}(j\omega)|^2 > 0 \Rightarrow \omega^4 + \left(\beta_1^2 - 2\bar{A}\right)\omega^2 + 2\left(\bar{C} - \bar{B}\beta_1\right) > 0 \tag{43}$$

Similar to theorem 3, if $\bar{C} - \bar{B}\beta_1 > 0$, the string stability is achieved which is equal to:

$$2\beta_1\alpha_2 - \left(a\beta_1 + 2\alpha_2 + 2\alpha_1\beta_1 - 2\alpha_2\beta_2\right)\beta_1 > 0$$
$$\Rightarrow \beta_2 > \frac{\beta_1\left(a + 2\alpha_1\right)}{2\alpha_2}$$

**Note 3**. If there is not any uncertain dynamics in the vehicle dynamical model, the adaptive rules (12) and (14) are put aside and the error equation (32) will be in the following form.

$$E_i(s) = \bar{\Gamma}_i(s)E_{i-1}(s) + \bar{\Gamma}_i(s)E_{i-1}(s), \tag{44}$$
$$\bar{\Gamma}_i(s) = \frac{a_i\left(\beta_2 + 1\right)s^2 + a_i\beta_1 s}{s^4 + \beta_1 s^3 + 2a_i\left(\beta_2 + 1\right)s^2 + 2a_i\beta_1 s}$$

Therefore, the condition (43) can be expressed as:

$$|\bar{D}(j\omega)|^2 - 4|\bar{N}(j\omega)|^2 > 0 \tag{45}$$
$$\Rightarrow \omega^4 + \left(\beta_1^2 - 2a_i\beta_2 - 2\bar{\upsilon}\right)\omega^2 - 2a_i\beta_1^2 > 0$$

The free term $-2a_i\beta_1^2$ is dominant in string stability analysis. Since this term is negative, the string stability is not satisfied. Therefore, adaptive terms play a major role in achieving string stability.

## 5. Simulation studies

Here, additional numerical results are presented to validate the introduced approaches of this paper. In particular, both internal and string stability of uncertain decentralized UD and BD vehicle convoys will be investigated. In all results, the distance error between vehicles *i* and *i*-1 is defined as $DE = p_{i-1} - p_i - l_{i-1} - R_{min}$ Two uncertain

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UD and BD convoys of 20 vehicles including a leading and 19 following vehicles are selected to implement both approaches theoretically. The constant parameters, adaptive gains and control parameters are considered as $c_D = 0.27,\ \rho = 1.07, l = 3.9m,\ R_{min} = 8m,\ \alpha_1 = 1.3,\ \alpha_2 = 2.2,\ \beta_1 = 1.2$ and $\beta_2 = 1.6$. Moreover, the leader velocity profile (m/s) is defined as follows

$$q_0(t) = \begin{cases} 20, & 0 \le t < 40 \\ t - 20, & 40 \le t < 60 \\ 40, & t \ge 60 \end{cases}$$

At first, the BD convoy is investigated. Fig. 2 illustrates the distance error of the convoy. In accordance with this figure, the *DE* asymptotically tends to zero which means the internal stability. Moreover, the maximum value of *DE* reduces along the convoy during accelerating the lead vehicle which indicates the string stability. Fig. 3 depicts the convoy velocity. Due to internal stability of convoy, each vehicle follows the velocity of the leader. Fig. 4 shows the estimates of uncertain dynamics $\sigma_{0,i}$ and $\sigma_i$, respectively.

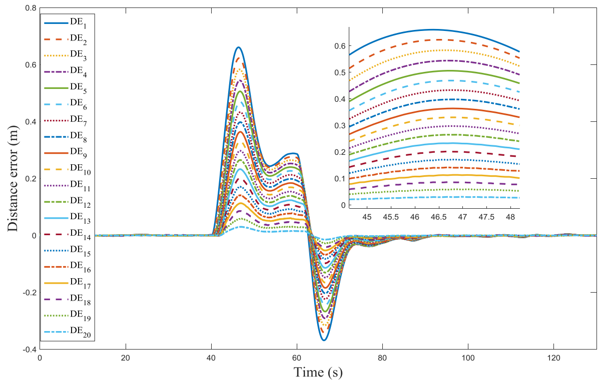


**Fig. 2.** Distance error of decentralized uncertain BD convoy.

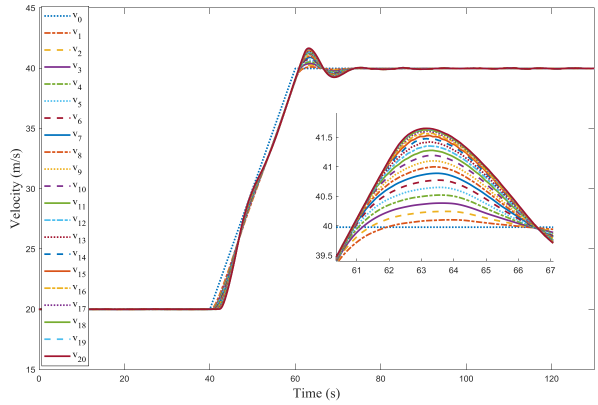


**Fig. 3.** Velocity of decentralized uncertain BD convoy.

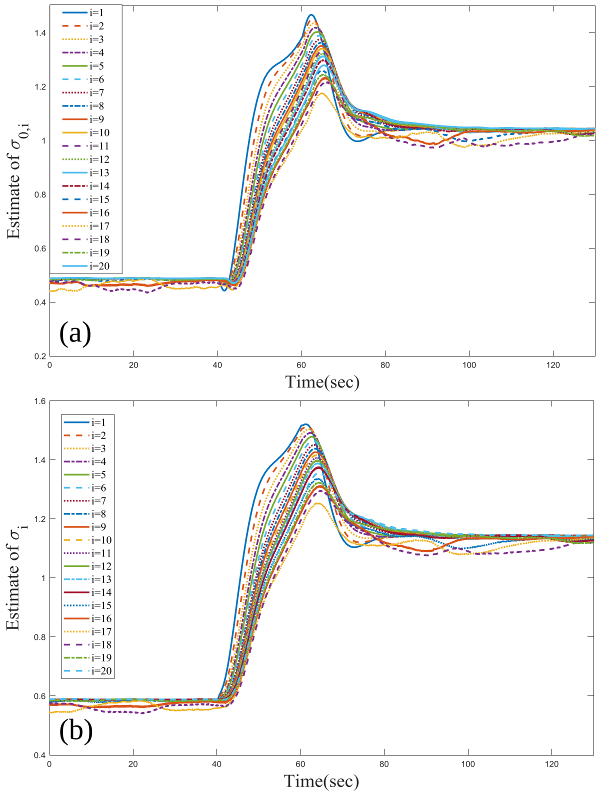


**Fig. 4.** Estimates of: a) $\sigma_{0,i}$ and b) $\sigma_i$ in BD topology.

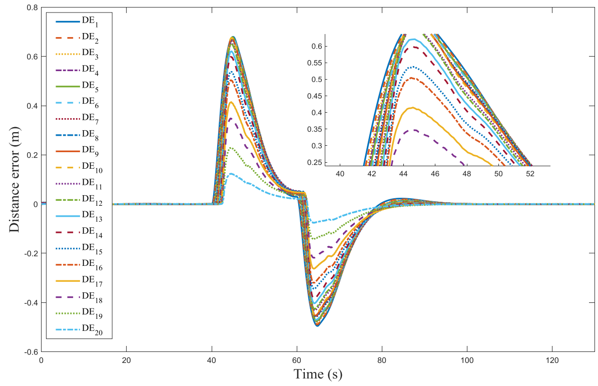


**Fig. 5.** Distance error of decentralized uncertain UD convoy.

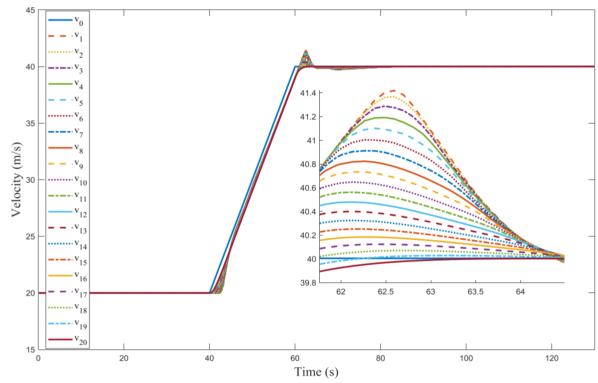


**Fig. 6.** Velocity of decentralized uncertain UD convoy.





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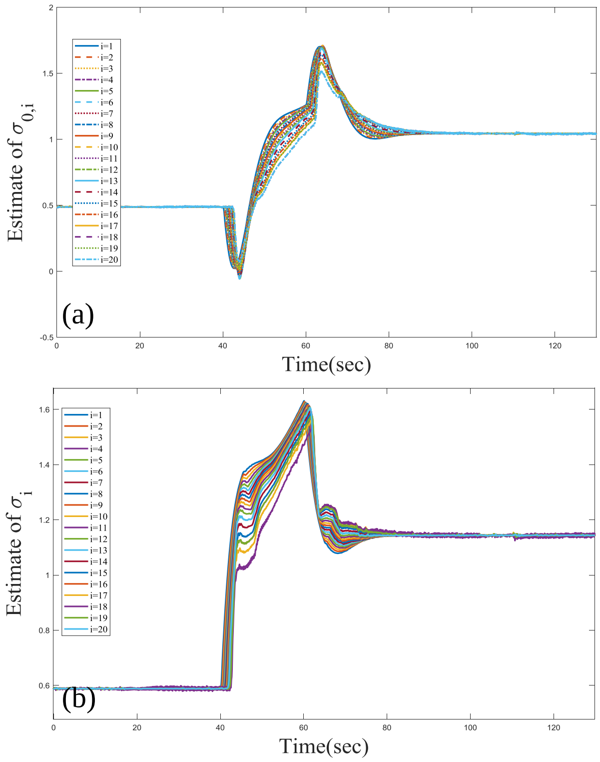


**Fig. 7.** Estimates of: a) $\sigma_{0,i}$ and b) $\sigma_i$ in UD topology.

Second, we study the uncertain UD vehicle convoy. In Fig. 5, the *DE* of the convoy is presented emphasizing both internal and string stabilities. The vehicles' velocities are plotted in Fig. 6. All vehicles asymptotically track the leader velocity. Moreover, the estimates of uncertain functions $\sigma_{0,i}$ and $\sigma_i$ are depicted in Fig. 7.

## 6. Conclusion

The stability problem of decentralized uncertain large-scale UD and BD vehicle convoys is investigated in this paper. It is assumed that only relative displacement information between consequent vehicles is accessible. The CDP is used to regulate the inter-vehicle distances. For each UD and BD, a robust adaptive size-independent control protocol is designed to achieve internal and string stability despite of existence of unknown dynamics by using only relative displacement information. The internal stability of UD and BD convoys are performed in frequency domain. According to the results, the control and adaptive parameters are tuned independently of the convoy size. Moreover, the additional constraints on control gains satisfying the string stability are calculated through error propagation analyses. Different simulation verifications are presented to illustrate the merits of both control techniques.

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