Experimental study of parameters and high-order values of velocity in the behind wake of a vehicle model

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Abstract

In this study an experimental analysis is conducted to explore the effects of the distance placement of a trailer model on the dynamic of flow and the higher-order parameters of velocity like flatness and skeweness in the wake of a Notch back car model. In addition, the changing trends of Strouhal number and the mixed length scale are depicted. All the experiments are conducted in the aerodynamic laboratory of Hakim Sabzevari University. In order to measure the property of flow, the researchers have made use of the wind tunnel device and hot wire anemometer produced by Farasanjesh-e-Saba Company. The results indicated that the values of skewness in the lower heights (near to ground) are less than their values in the upper heights (near to roof of model) and this issue is in contrast as the distance from the car increases. The values of flatness also gradually decrease by an increase in the distance from the car. The Strouhal number often reduces by increase in the distance of car from trailer and the mixed length scale in the width of trailer often has one or two maximum peaks. The wake of trailer in positions near the car is not effective in the formation of maximum peaks of mixed length scale sites.

Keywords: Car model, wake flow, skewness and flatness, hot wire anemometer

1. Introduction

In spite of the fact that turbulence is one of the very old issues in the domain of fluid mechanics, it has been remained unresolved. This phenomenon generally exists in most of the issues related to energy transformation, fluid flow, transmission systems, etc. [1]. The possible method for the description of turbulence, with the help of general laws of continuous mechanics, was established by Reynolds at the end of the previous century. In this method, the field velocity of turbulence is decomposed into two main components: one of them is related to the average motion and the second one to the time-dependent fluctuations of fluid velocity. Therefore, the most logical method for describing turbulence is related to the theories which are formed based on the statistical hypotheses related to dynamic equations of fluid flow. The statistical theory requires some information about the investigation of the probability density distribution function and the corresponding correlation functions. The higher level co-relational functions (like skewness and flatness …) lead to the improvement of comprehensiveness of the specific turbulent model in terms of statistics.

Homogeneous Isotropic Turbulence (HIT) with a negative skewness determined in the velocity differentiation of in which is the fluctuating component of velocity across . Skewness defined as follows [2]:

$$S_u = 1/N \sum_1^N \frac{(u(n) - \overline{U})^3}{\sigma_u{}^3} \quad (1).$$

In Homogeneous Isotropic Turbulence, Skewness is indicative of the extent of eddy production through eddy expansion and its zero values are derived from the nonlinear equation of Navier- Stokes. The theoretical predictions for Skewness and its dependent to Reynolds' turbulent number (in which is fluctuation of velocity, is micro-scale of tailor and is kinematic Viscosity) has been widely considered in the past. While Kolmogrov [2] has argued that S should be kept constant, other researchers based on the subsequent discussions of Kolomogrov [3] have maintained that with an increase in Re, there should be an increase or decrease in S [4].

The hot wire anemometer can provide a precise measurement of S and a solution which is not

matched with other experimental methods. Despite this fact, the three-dimensional clarity of Probe has a negative effect on the estimation of S. In fact, the velocity derivation is influenced by the Small Scale Motion (SSM) which is difficult its correct analysis. In another way, the clarity of timed sampling of the velocity signal affects the measurement of SSM. By using direct numeric simulation of turbulence state, we can analyze the resultant effects of limited space and the provisional clarity of the utilized hot wire anemometer in experiments.

The main idea for conducting this endeavor is derived from several articles especially [5] in which by numerical estimation of statistical parameters like the third and fourth moments and their spatial derivation, the changes in these components on the dynamics between the two turbulent flows are explored. The main objective of the present study is investigating the trends of changes in the high-order parameters of velocity, eddy shedding frequency Strouhal numbers and the mixed scale length.

2. PREVIOUS RESEARCH

There has been a lot of experimental and numerical research on different models of cars so far. As for some of these studies, are reference can be made to a study conducted by Ahmed [6] in which he has considered a simple model like a car and by experimenting different models with different angles of the behind mirror he has recorded the results. Some of the researchers like [7-8] have numerically simulated similar models. Since the recorded results by Ahmed are rather limited, the criterion for comparing these numerical works is generally bound to the changes in the drag coefficient. Precise experiments on this model are conducted by [9-10] and different experimental results like velocity vectors, Reynolds tensions, etc. are measured and recorded by LDA. Khalighi et al., [11] in a set of experiments reached to intensity differentiation curves and velocity curves for a model of car. Sadafian et al., [12] analyzed the effects of a number of parameters in the calculation of coefficients of drag. Shayesteh et al., [13] attempted to numerically study a model of car. In addition, Javarkashian et al., [14] attempted to conduct experimental studies on this model. Watkins et al., [15] investigated the changes in the coefficients of drag and lift in Ahmed's model in a subsequent fashion.

Turbulence will be distributed by pressure fluctuations and velocity in the field. Statistical properties are sensitive to anisotropy and for the analysis of this anisotropy we can make use of

statistical quantities. This anisotropy can be related to the turbulence and therefore with an increase in the mixture, the anisotropy increases and some flow parameters like the third and fourth normal statistical moments (skewness and kurtosis) increase as well.

A total review of the statistical relations in high-order moments of velocity in the inside channel flows, the border layer and free jet are presented in [16]. Wirvali and Raht [17] conducted an experimental study and indicated the statistical properties like skewness and kurtosis in the mixed layer distance from the normal distribution. They observed that the maximum point of skewness and kurtosis is not in the center of the mixed layer. The results indicated that this maximum point is near a region with a lower energy. This issue indicates that mixture along with the penetration of eddies of higher energy region occurs towards the low level energy region. In 2008, Kong and Menovi [18] reached similar results as those found by Wirvali and Raht, but the Reynolds was higher and by the simulation model of large eddy, they numerically indicated that the effects of large eddies on skewness and kurtosis is more than small eddies. Matovic used Laser-Doppler anemometer for measuring higher order velocity moments in the free flow of jet [19].

The collected results inspired the researchers to conduct similar experiments in the free flows of jet [20]. For this purpose, they made use of hot wire flow probe X for measuring velocity moments up to the sixth level moments and analyzed the kurtosis and skewness values. Johnson et al., [21] analyzed velocity turbulence parameters in the range of Reynolds numbers 2530<Re<27300 for outer layer with eddy pressure of zero. They conducted their studies by using hot wire anemometer and hot film for measuring the wall shear tension and the effects of changes in Reynolds number and the size of used probe on the intensity of turbulences, skewness and flatness of the flow.

Since different parameters of car's wake like skewness, strouhal number and mixed length scale are not experimentally investigated, the present study attempts to analyze the change trends in these parameters in details

3. Laboratory Equipment

Measuring the characteristics of flow in this experimental endeavor was performed by using hot wire anemometer and in a low velocity wind tunnel. The applied wind tunnel device had a test section with a length of 168 cm and a width and height of 40 cm. The appropriate design of the tunnel leads to the creation of free flow turbulence intensity of about

1.0% in the direction of the flow and the intended device has a high accuracy. The maximum velocity is 30 m/s. The inflow in this experiment is 10 m/s. The utilized hot wire anemometer was in the form of a fixed flow which has a one component probe. The hot-wire anemometer system is equipped with an external calibrator. Hot wire signals with 5KHZ sampling frequency and with using the recorded data is saved. The hot wire flow meter probe is one-dimensional and the Sensor is made of tungsten with 5 micron diameter. The wind tunnel device and the hot wire anemometer used in the present study were produced by Fara sanjesh-e-Saba Company. In Fig. 1, there is a view of the intended wind tunnel and its installed model.

The experimented models were a simplified model of a Notch back car model and an eight class trailer model without accessories like mirrors, antenna, wheels, etc. Considering the Blockage coefficient of determination as an important parameter is the first point in creating the model. In order to ignore the effect of fluid flow on the lateral walls of the test section on the surface of model, the suggested values for the Blockage coefficient were from 0.05 to 0.1. In this research, the selected value for this coefficient, based on the experimental conditions and the wind tunnel, is 0.09 based on which the scale of models is $\frac{1}{75}$. In these experiments, at first, the car model was

independently exposed to the flow and then it is positioned in distances of 0.01, 1, 2, 3 and 4 times the length of trailer and behind it and in each case data gathering is conducted in the distances of 0.01, 0.25, 0.50, 0.75, 1, 1.25 and 1.75 times the length of the car and its behind (Fig. 2). Static pressure changes in the experiment chamber are fixed based on the diffuser design and the experiments are conducted internally.

4. Validation

At first, for ensuring the functionality of the wind tunnel channel and the hot wire anemometer, a sample of data were gathered and compared with the work of other scholars. Since no previous studies had investigated the selected flat wake explored in the present study, a cubic cylinder model was used. The average time component diagram of velocity in the dominant direction of flow (U) for a cubic cylinder type with the ratio of $b/h = 1$ (b is the width and h is the height of cylinder) and in the Reynolds of 8600 in two different time periods are depicted in Fig. 3.

As it is depicted, rather good comparable results are found between the findings of the present study and those of Saha et al. [22] and Shadaram et al. [23] which approximately have rather similar Reynolds number. (It is worth-mentioning that this point is brought here to validate the flow results.)
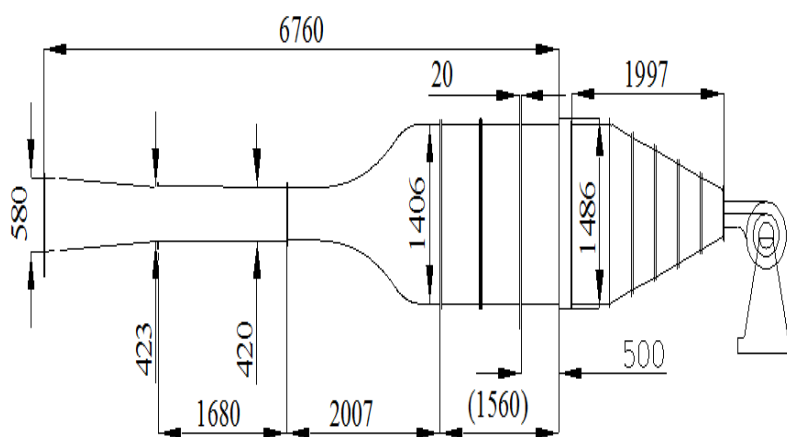

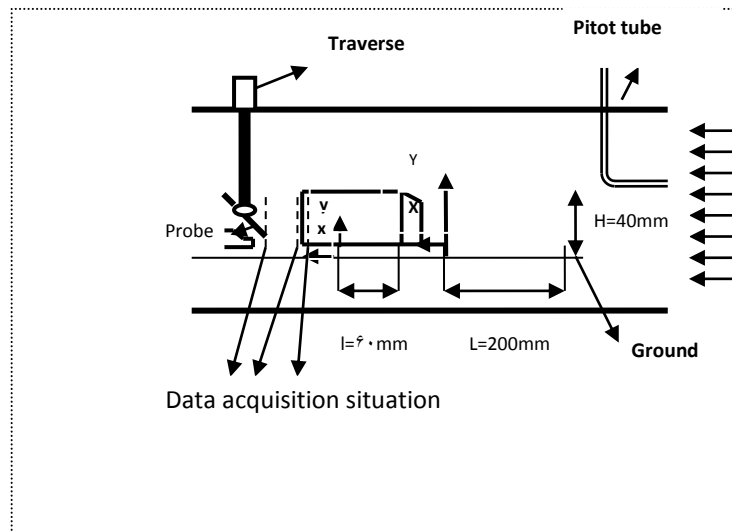**Fig1.**. Schematic view of the wind tunnel (in mm).

Fig2. Schematic view of the models inside of the tunnel and data collection mechanisms
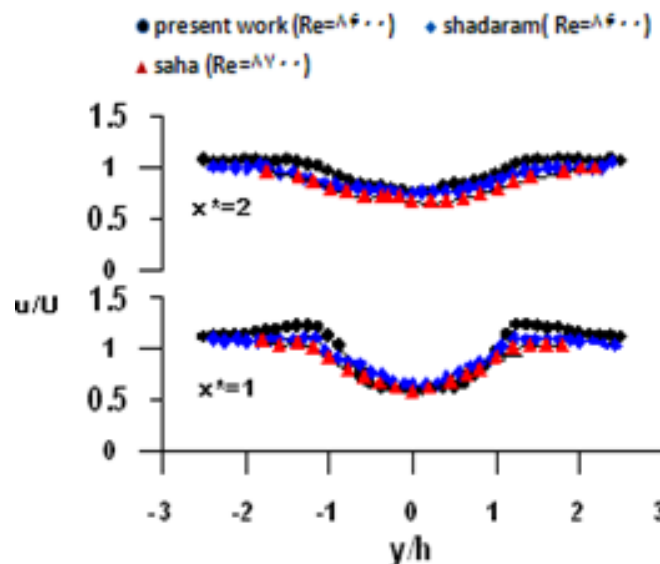

Fig3. Average velocity profile for square cylinder in two different time periods in details.

5. RESULTS AND DISCUSSION

Anisotropy in the high-order moments can be identified better. Therefore, for observing the anisotropy of the field, the third and fourth normal moments known as skewness (skewness coefficient) and kurtosis (flatness coefficient) are used. Skewness and kurtosis are statistical parameters which are used in the qualitative analysis of the fluid flow. The turbulent velocity component, U, which is in the inhomogeneous direction of the field, has the duty of transferring motion energy from passing the mixed layer. This component, in the process of mixture, also creates anisotropy in mixed layer. The values of normal moments, which change with time, indicate the extent of anisotropy. Skewness distribution is the main instrument in identifying the extent of period and anisotropy. In fact, skewness is a criterion for determining presence or lack of period in the distribution. For a completely symmetrical distribution, the skewness of zero and for an asymmetrical distribution with a tendency towards the higher values, the positive skewness and for asymmetrical distribution with tendency towards lower values, the value of skewness is negative which is indicative of the amount of symmetry of instantaneous velocity relative to the average velocity

of the fluid flow. When the skewness is something other than zero, the data do not have asymmetry relative to the average value and it is not possible to model the data by a normal distribution. The fluctuations of the horizontal component of the velocity and their degree and intensity are rather small. Kurtosis has been normal based on the fourth transformation. In other words, kurtosis is a criterion for the sharpness of the curve in the maximum point. The value of kurtosis for normal distribution is equal to 3. These parameters are defined as follow:

$$S_u = \frac{1}{N} \sum_1^N \frac{(U(n) - \overline{U})^3}{\sigma_u^3}$$

$$K_u = \frac{1}{N} \sum_1^N \frac{(U(n) - \overline{U})^4}{\sigma_u^4} \qquad (2,3)$$

In reality, oscillogram indicates the instantaneous velocity and most of the data in that region are higher than the average velocity. However, there are negative jumps which reduce the average velocity. The instantaneous jumping value can be identified by calculating the value of skewness. In Fig. 4-6, it is observed that most of the values of skewness are against zero and have positive and negative values which are indicative of an anisotropic turbulent field in the wake. It is observed in Fig. 4 that in the most positions of the car, in the wake and in the isolated state of the car, in lower (near to ground) height of the wake, the values of skewness are often negative which is indicative of this fact that the instantaneous velocity is lower than average velocity and has maximum jumps. But in the upper heights (near to roof of model), the values of skewness are positive and the instantaneous velocity is higher than the average velocity and has fewer minimum jumps compared to it. In the lower and upper heights, the wake peaks of maximum skewness are present and in these parts the extent of mixture is higher and the differences in the values of instantaneous velocity with the average velocity are more than the other parts in the wake. In the middle parts of the wake for the isolated and other positions of the car in the back of trailer it is observed that the values of skewness have gradually developed from negative to positive and in the three secondary positions are rather similar and equal to each other. The worth-mentioning point is that by locating the car at the back of the trailer, in the middle parts of the wake, the values of skewness are positive which is indicative of the fact that often the values of the instantaneous velocity are higher than the average velocity, but it has fewer minimum peaks.

In Fig. 5, we can see that for the isolated condition in lower height the values of skewness are still negative, but in the middle and upper heights of the behind wake the values of skewness have become positive. In fact, in this range, the values of instantaneous velocity are higher than the average velocity and have fewer minimum peaks than the average velocity. Except for the x/L=0 position, in other positions of the car behind the trailer the values of skewness are positive and, in fact, we observe an asymmetrical distribution towards higher values. Gradually and by distance from the data gathering position from the car (Fig. 6), it is observed that in all the condition of car position, the values of skewness in the wake are positive. Another worth mentioning point is that with an increase in the wake of the car, except for x/l=0 conditions which has a different process, the skewness in other conditions has a downward condition.

Kurtosis is the same as the fourth central moment. Kurtosis refers to the extent of stretch or flatness of Probability Density Distribution Curve relative to the Gaussian Density Function. In fact, the extent of the sudden peak of the probable distribution curve to the normal distribution is expressed by the extent of flatness. Accordingly, the positive or negative flatness are indicative of the more stretchiness or flatness of the density curve to the Gaussian density distribution, respectively (Fig. 7). In a case where the extent of flatness is positive, the sequences of data distribution and the changes in the domain of data relative to the time are higher. As it was depicted in Fig. 7, in Leptokurtic curve the flatness is positive. In this condition, the sequence of data distribution is lengthy and the changes in the domain of data relative to the time are higher. Therefore, the instantaneous velocity in most of the cases distances from the medium velocity and the jumping and turbulences of the instantaneous velocity flow become higher. In the Platykurtic curve, the flatness is negative and compared to the Gaussian density distribution, the instantaneous velocity is lower than the medium velocity, but the sequences of density distribution are bulkier.

Therefore, in this state, the instantaneous velocity in most of the cases becomes more distanced from the medium velocity and the jumping and turbulences of the instantaneous velocity of the flow increase. For the positive flatness, the instantaneous velocity compared to the distribution state of Gaussian is lower, but the sequences of density distribution become bulkier.

Fig.8 to 10 shows the process of changes in the kurtosis (flatness coefficient) or the fourth central moment. In Fig.8, most of the values of kurtosis are

positive and the probable density distribution curve is more stretched than the Gaussian density distribution. The instantaneous velocity also in most of the cases distances more from the medium velocity and the jumping and turbulences of the instantaneous velocity of the flow increase. The stretch of the probable density distribution curve for the x/L=0 and x/L=4 states in the lower height of the wake is more tangible and we can observe the maximum peaks of kurtosis. The distances of data gathering positions from the car (Fig.9) indicates that for the isolated mode the values of kurtosis x/L=4 and x/L=3 become equal to zero which is indicative of the equality the probable density distribution curve with the normal distribution of Gaussian density. As the distances of most data gathering states from the car increases (Fig.10), it is observed that the probable density distribution curve

for the isolated mode is still equal to the normal distribution of Gaussian density. However, in other positions of the car in the trail the values of kurtosis are positive. In addition, the sequences of data distribution are lengthy and the changes in the domains of data relative to the time are higher. Except for the x/L=0 state, in other cases as the height increases, the values of kurtosis experience a downtrend and the probable density distribution curve becomes closer to the Gaussian density distribution. In x/L=0 state the process is quite the reverse. In lower heights, the density distribution curve is equal to the Gaussian density distribution curve and by an increase in the height, the stretch of curve increases and become more distanced from the Gaussian density curve.
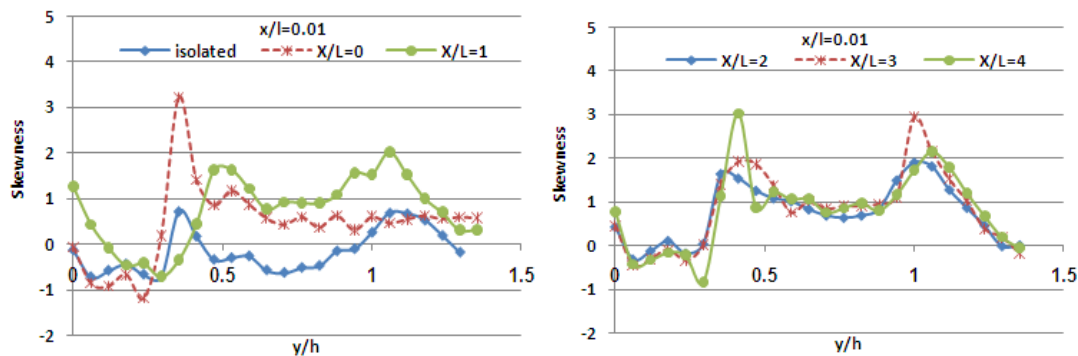

Fig4. The trend of change in skewness for different positions of car in the flat wake of the trailer for x/l=0.01
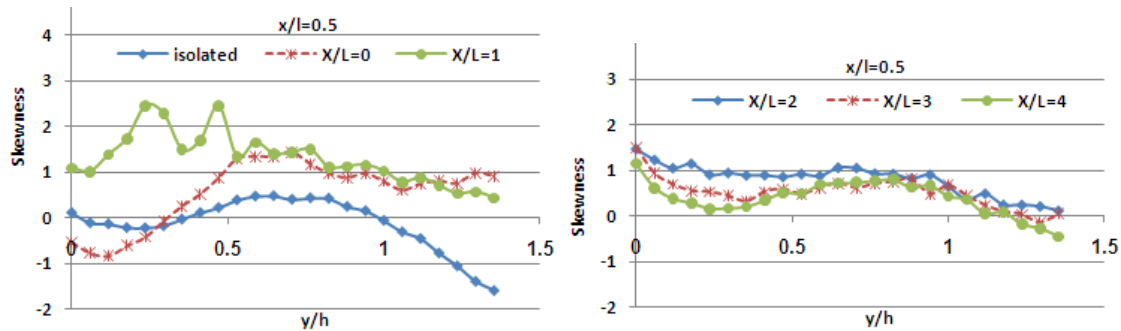

Fig5. The trend of change in skewness for different positions of car in the flat wake of the trailer for x/l=0.5
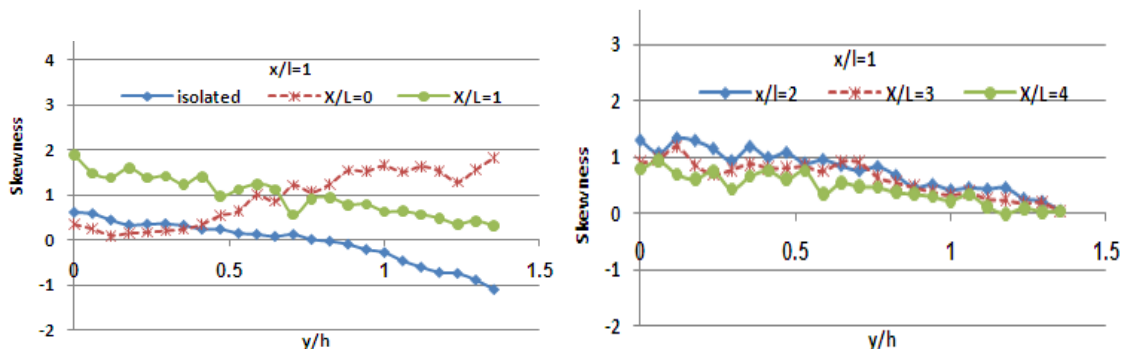

Fig6. The trend of change in skewness for different positions of car in the flat wake of the trailer for x/l=1
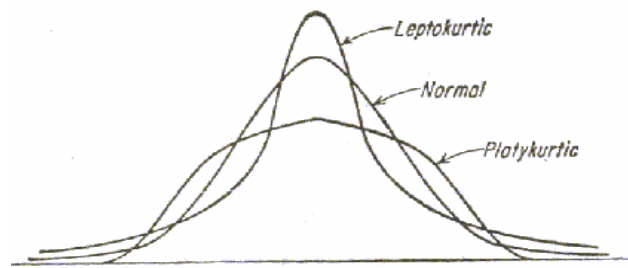
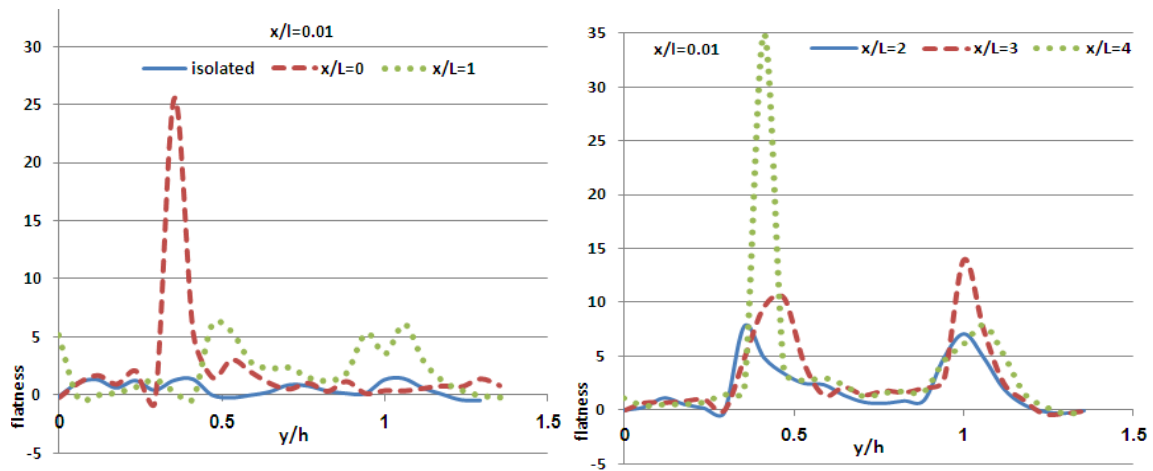**Fig7.** The two states of flatness and stretchiness relative to the normal state

Fig8. The trend of changes in flatness coefficient in different positions of the car in the wake trailer for x/l=0.01
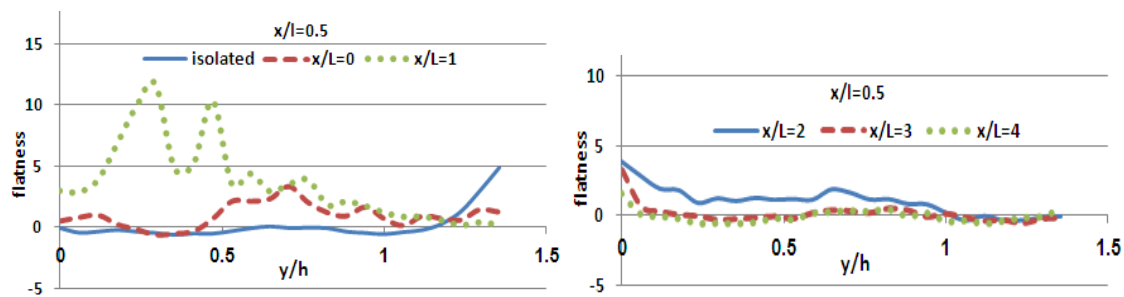

Fig9. The trend of changes in flatness coefficient in different positions of the car in the wake trailer for x/l=0.5
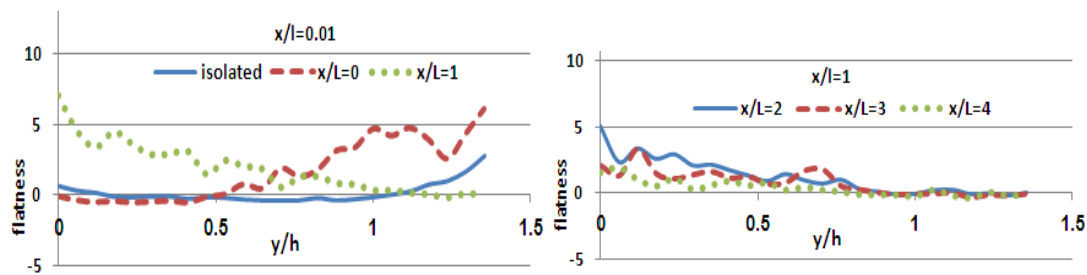

Fig10.   The trend of changes in flatness coefficient in different positions of the car in the wake trailer for x/l=1

Strouhal number is a dimensionless number which presents the frequency of Carman vortex oscillations formed in the back of the model as a dimensionless value. It is defined as follows:

St=f*h/Wref   (4).

Where f, h and Wref are the frequency of vortices at the back of the model, model high and velocity of free fluid flow respectively. The frequency of vortices formed at the back of the model could be calculated with the sensor of hot wire anemometer in wind tunnel.

Fig. 11 indicates the changes in Strouhal numbers in the back of car for the different positions of the car on the trailer. The analyses of the diagrams reveal that for the isolated model and the state in which the car has the position of X/L=1, the maximum and minimum peaks are clearly observable. In other positions, with a distance from the car, the values of Strouhal number gradually decreases which is clearly observable for X/L=0 position. In X/L=2 position, the Strouhal number by getting away from the model decreases with a very gentle slope.

A suggested method for identifying the Reynolds' shear stress amount is relating the amount of this turbulent stress to the medium velocity field. This method has performed successfully in describing the free turbulent flows like wake and jet. The success of this method is limited to the two dimensional issues. [1]

Prandtel [24] indicated that   in which   is the length of the mixed scale and a criterion for measuring eddies. In a way, based on Goledston's [25] analysis: by putting   in the above equation we can calculate the length of the mixed scale.

Fig. 12 indicates different values of the length of the mixed scale for different distances of the car position in the wake of trailer. As it is observed, the largest length of the mixed scale in the wake for different positions of the isolated, X/L=0 and X/L=1 models shapes in the x/l=0.75 position behind the car. In the isolated and X/L=1 positions, the maximum lengths of the mixed scale are much closed to each other. By increasing the distance of the car with the trailer, we can observe the formation of two mixed scale length maximum peaks in the two x/l=0.25 and x/l=1.25 for X/L=2 positions. With an increase in the distance of the car from the trailer, the length of mixed scale becomes smaller and closer to each other. A worth-mentioning point is that exactly in the position in which we observed the maximum length of the mixed length scale for different positions of the car; we have the minimum peaks of mixed length scale for the two subsequent positions. As the car gets more away from the trailer, another maximum peak of the mixed scale length is formed which is more distanced from the three initial positions. It was also observed that the wake trailer for the two positions of X/L=0 and X/L=1 does not have an effect on the formation of maximum peaks of the mixed length scale.
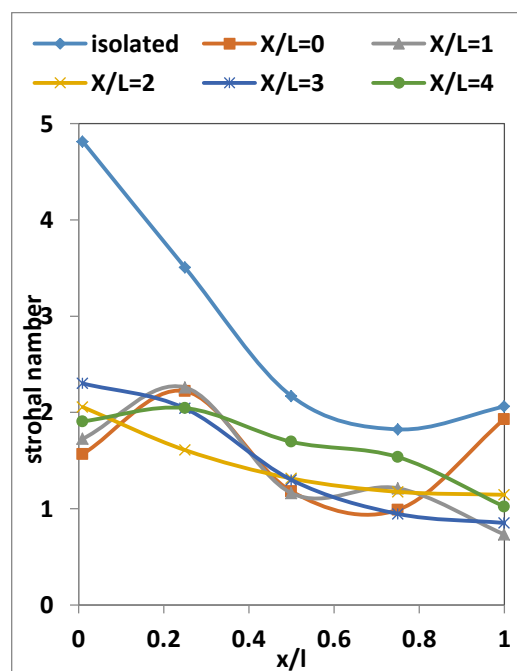

Fig11.   The trend of changes in Strouhal number for different positions of the car in the wake trailer
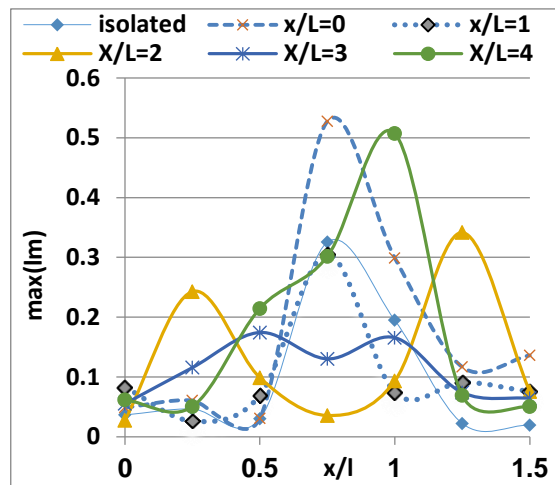
Fig12.   The process of changes in the mixed scale length for different positions of the car in the wake trailer

6. CONCLUSIONS

In the present study, the existing statistical parameters between the higher order velocity values, namely skewness and flatness coefficients, in the behind wake of a car model are investigated. For this purpose, the hot wire anemometer was used to measure the characteristics of the flow and the related parameters of the turbulent components of velocity in different positions of the behind wake of the car.

Some of the findings of the present study are as follow:

Large changes in the higher order moments close to the Gaussian values statistically leads to higher complexity of the wake flow arrangement compared to the free jet flow and other limited flows.

There are maximum peaks of skewness in the lower and upper heights of the wake and in these parts the extent of mixture is higher.

In most of the cases, increase in the height of the wake of the car results in a decreasing level of skewness.

In other cases, with an increase in the height, the kurtosis values have a downward decreasing trend and the probable density distribution curve becomes closer to the Gaussian density curve. In x/l=0.01 state the process is the reverse, and in the lower heights the density distribution curve is equal to the Gaussian density distribution curve and with an increase in the height, the curve becomes more stretched and gets away from the Gaussian density curve.

The wake of trailer for the close positions of the car to the trailer does not have an influence on the place position of the formation of the maximum peaks of the mixed length scale.

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